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# VISCOSITY OF A DILUTE SUSPENSION IN A HIGH-FREQUENCY ELECTROMAGNETIC FIELD

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*The action of a high-frequency electromagnetic field on a dilute suspension of spherical particles with a constant dipole moment is studied using statistical mechanics. An expression for effective viscosity is obtained. It is shown that the shear viscosity of the dilute suspension depends on the frequency, magnitude, and direction of the high-frequency electromagnetic field. Depending on the frequency of the high-frequency electromagnetic field, the rotation of the suspension particles is decelerated or accelerated, with the viscosity increasing or decreasing, respectively. It is shown that the acceleration of the suspension particles by a high-frequency electromagnetic field (and, hence, the decrease in shear viscosity) has a resonant nature.*

The behavior of suspensions in quasistationary electromagnetic fields has been the subject of much investigation [1–4]. These studies, however, are not complete because they cover a narrow part of the electromagnetic radiation spectrum. It is of interest to study the behavior of suspensions in electromagnetic fields of the high-frequency range ($10^6$–$10^9$ Hz), which have a number of distinguishing features. It is necessary to take into account that thermodynamic and hydrodynamic quantities, such as fluid velocity, density, temperature, pressure, etc., vary much more slowly than electric and magnetic intensity vectors. This means that the oscillation period of a high-frequency electromagnetic field (HFEMF) is much smaller not only than the characteristic time of the problem $t_0 = L/u$ ($L$ and $u$ are the characteristic dimension and velocity of the problem) but also than the characteristic time during which the mechanism of phenomena such as viscosity and thermal conduction change. Hence, the thermohydrodynamic state of a small element of the medium cannot change significantly over the oscillation period of the HFEMF. Therefore, apparently, the state of the medium is reasonably characterized by thermodynamic and hydrodynamic quantities averaged over the oscillation period of the HFEMF. It should be noted that studies in this frequency range as applied to suspensions practically have not been performed.

In the present work, the results of [1], obtained for dilute suspensions in quasistationary electromagnetic fields, are extended to the case of high-frequency electromagnetic action.

We consider a homogeneous HFEMF. This means that the wavelength in the medium should be larger than the characteristic dimensions of the problem: $\lambda \gg L$ ($\lambda = c\sqrt{\varepsilon'}/\nu$ is the wavelength, where $c$ is the velocity of light in vacuum, $\nu$ is the HFEMF frequency, and $\varepsilon'$ is the material's permittivity). This leads to the following restriction on the HFEMF frequency: $\nu \ll c\sqrt{\varepsilon'}/L$. Thus, for channel dimensions of about 0.1 m, $\nu \ll 10^9$ Hz and $\nu \approx 10^6$–$10^8$ Hz.

The flow is assumed to be isothermal, i.e., the heat sources that arise during the action of the HFEMF on the dilute suspension are ignored.

Let us consider a suspension of spherical particles which possess constant dipole moment $\mu$. For definiteness, we assume that the particles have an electric dipole moment and, hence, undergo the action of the electric intensity vector. With a corresponding change of notation, the results obtained are also valid for a suspension of particles with a magnetic dipole.

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In view of the aforesaid, the homogeneous HFEMF can be written as [1, 5]

$$\boldsymbol{E} = E_0\boldsymbol{h} = E_0 \exp{(i\omega t)}\boldsymbol{h}_0, \tag{1}$$

where $E_0$ is the amplitude of the electric intensity vector, $i$ is imaginary unity, $\omega$ is the circular frequency of the HFEMF, $t$ is time, $\boldsymbol{h} = \boldsymbol{h}_0 \exp{(i\omega t)}$ is a vector in the field direction, and $\boldsymbol{h}_0$ is a unit vector directed along the axis about which the HFEMF intensity vector oscillates. The angles defining field directions in spherical coordinates are given by the formulas

$$h_{01} = \cos\psi \sin\theta, \qquad h_{02} = \sin\psi \sin\theta, \qquad h_{03} = \cos\theta, \tag{2}$$

where $\psi$ and $\theta$ are the latitude and longitude, respectively.

According to [1], the stress tensor is given by

$$\sigma_{ik} = -p\delta_{ik} + 2\eta_0(1 + 5\varphi/2)\gamma_{ik} + n\mu(\overline{\langle e_i\rangle E_k} - \overline{\langle e_k\rangle E_i})/2. \tag{3}$$

(It should be noted that in [1] quasistationary fields are considered, and, hence, as was noted above, the expression for the stress tensor must be averaged over the HFEMF oscillation period.) In (3), the bar denotes averaging over the HFEMF oscillation period, $p$ is the pressure, $\eta_0$ is the shear viscosity of the fluid in which the particles are suspended, $\varphi$ is the volumetric fraction of the solid phase, $\gamma_{ik} = (\partial v_i/\partial x_k + \partial v_k/\partial x_i)/2$ is the symmetric velocity-gradient tensor, $v_i$ is the fluid velocity, $\delta_{ik}$ is the Kronecker delta, $n$ is the number of particles, and $\langle e_k\rangle$ is the first-order moment of the distribution function.

In the case of dipole spherical particles, the moments of the distribution function are given by the equations [1]

$$\frac{d\langle e_k\rangle}{dt} = -\frac{1}{\tau_1}\langle e_k\rangle + \omega_{kj}\langle e_j\rangle + D\text{æ}(h_k - \langle e_k e_j\rangle h_j); \tag{4}$$

$$\frac{d\langle e_i e_k\rangle}{dt} = -\frac{1}{\tau_2}\Big(\langle e_i e_k\rangle - \frac{1}{3}\,\delta_{ik}\Big) + \omega_{ij}\langle e_j e_k\rangle + \omega_{kj}\langle e_j e_i\rangle$$

$$+ D\text{æ}(\langle e_i\rangle h_k + \langle e_k\rangle h_i - 2\langle e_i e_k e_j\rangle h_j), \tag{5}$$

etc. Here $\tau_\alpha = 1/(\alpha(\alpha+1)D)$ are the relaxation times ($\alpha = 1, 2, \ldots$), $D$ is the coefficient of rotational diffusion, $\omega_{ik} = (\partial v_i/\partial x_k - \partial v_k/\partial x_i)/2$ is the antisymmetric velocity-gradient tensor, $\text{æ} = \mu E_0/(kT)$, $k$ is the universal Boltzmann constant, $T$ is the absolute temperature of the suspension, and $\langle e_i e_k\rangle$ and $\langle e_i e_k e_j\rangle$ are the second- and third-order moments of the distribution function, respectively.

To solve system (1), (3)–(5), we use the fact that the relaxation times decrease with increase in the moment number ($\tau_1 > \tau_2 > \tau_3 > \ldots$). Hence, for any motion, it is possible to find a number such that the corresponding moment can be set equal to its equilibrium value in the given field. Next, Lower order moments can be calculated. The simplest approximation can be obtained if the second-order moment is set equal to its equilibrium value in the HFEMF. Then, the equilibrium values of the first $\langle e_i\rangle_0$ and second $\langle e_i e_k\rangle_0$ order moments of the distribution function are written as [1]

$$\langle e_i\rangle_0 = L_1 h_i, \qquad \langle e_i e_k\rangle_0 = (1 - L_2)\delta_{ik}/2 + (3L_2 - 1)h_i h_k/2, \tag{6}$$

where $L_1 = \coth\text{æ} - \text{æ}^{-1}$ is the Langevin function and $L_2 = 1 - 2\text{æ}^{-1}L_1$. From this we obtain $\langle e_k e_i\rangle_0 h_i = h_k - 2\text{æ}^{-1}\langle e_k\rangle_0$. Substituting this expression into (4), we obtain the relaxation equation as a first approximation:

$$\frac{d\langle e_k\rangle}{dt} = -\frac{1}{\tau_1}(\langle e_k\rangle - \langle e_k\rangle_0) + \omega_{kj}\langle e_j\rangle. \tag{7}$$

Thus, to calculate the stress tensor (3), it is necessary to know the first-order moment, which is defined by Eq. (7).

For the HFEMF $\boldsymbol{E}$ (1) with $\langle e_i\rangle_0 = L_1 h_i$ (6) and $\boldsymbol{h} = \boldsymbol{h}_0 \exp{(i\omega t)}$, the solution should be sought in the form $\langle e_k\rangle \sim \exp{(i\omega t)}$. Then, Eq. (7) is written as

$$i\omega\langle e_k\rangle = -(\langle e_k\rangle - \langle e_k\rangle_0)/\tau_1 + \omega_{kj}\langle e_j\rangle. \tag{8}$$

If only terms of the first order in $\omega_{kj}$ are taken into account, the solution of Eq. (8) has the following form (subscript 1 is omitted):

$$\langle e_k\rangle = \langle e_k\rangle_0/(1 + i\omega\tau) + \omega_{kj}\tau\langle e_j\rangle_0/(1 + i\omega\tau)^2.$$

Next, substituting this solution into expression (3), we perform averaging over the HFEMF oscillation period using the method proposed in [5]. For this, we separately consider the term

$$\overline{\langle e_i\rangle E_k} = \frac{\langle e_i\rangle + \langle e_i\rangle^*}{2}\,\frac{E_k + E_k^*}{2} = \frac{\langle e_i\rangle^* E_k + \langle e_i\rangle E_k^*}{4}$$

$$= \frac{L_1}{4}\Big(\frac{h_i^* h_k E_0}{1 - i\omega\tau} + \frac{h_i h_k^* E_0^*}{1 + i\omega\tau} + \omega_{ij}\tau\Big(\frac{h_j^* h_k E_0}{(1 - i\omega\tau)^2} + \frac{h_j h_k^* E_0^*}{(1 + i\omega\tau)^2}\Big)\Big)$$

(asterisk denotes complex conjugation).

The general expression for the stress tensor averaged over the HFEMF oscillation period becomes

$$\sigma_{ik} = -p\delta_{ik} + 2\eta_0\Big(1 + \frac{5}{2}\,\varphi\Big)\gamma_{ik} + \frac{1}{8}\,n\mu L_1\Big(\frac{h_i^* h_k E_0}{1 - i\omega\tau} + \frac{h_i h_k^* E_0^*}{1 + i\omega\tau} - \frac{h_k^* h_i E_0}{1 - i\omega\tau} - \frac{h_k h_i^* E_0^*}{1 + i\omega\tau}$$

$$+ \omega_{ij}\tau\Big(\frac{h_j^* h_k E_0}{(1 - i\omega\tau)^2} + \frac{h_j h_k^* E_0^*}{(1 + i\omega\tau)^2}\Big) - \omega_{kj}\tau\Big(\frac{h_j^* h_i E_0}{(1 - i\omega\tau)^2} + \frac{h_j h_i^* E_0^*}{(1 + i\omega\tau)^2}\Big)\Big). \tag{9}$$

For the HFEMF $\boldsymbol{E}$ (1) with the properties $h_{0j} = h_{0j}^*$, setting $E_0 = E_0^*$ and taking into account that for spherical particles, $\tau = 3\varphi\eta_0/(nkT)$, we write the real part of expression (9) in the form

$$\sigma_{ik} = -p\delta_{ik} + 2\eta_0\Big(1 + \frac{5}{2}\,\varphi\Big)\gamma_{ik} + \frac{3}{4}\,\varphi æ L_1\,\frac{1 - \omega^2\tau^2}{(1 + \omega^2\tau^2)^2}(\omega_{ij}h_{0j}h_{0k} - \omega_{kj}h_{0j}h_{0i}).$$

Let us consider simple shear motion ($\nu_{12} \neq 0$ and $\nu_{ij} = \partial v_i/\partial x_j$ is the velocity gradient tensor) with an arbitrary direction of the HFEMF oscillations. The effective shear viscosity is given by

$$\eta = \eta_0 + \eta_0\varphi\Big(\frac{5}{2} + \frac{3}{4}\,æ L_1\,\frac{1 - \omega^2\tau^2}{(1 + \omega^2\tau^2)^2}(h_{01}^2 + h_{02}^2)\Big).$$

Introducing the angles defining the HFEMF oscillation directions by formulas (2), we finally obtain

$$\eta = \eta_0 + \eta_0\varphi\Big(\frac{5}{2} + \frac{3}{4}\,æ L_1\,\frac{1 - \omega^2\tau^2}{(1 + \omega^2\tau^2)^2}\sin^2\theta\Big). \tag{10}$$

An analysis of expression (10) shows that depending on the HFEMF frequency, the effective shear viscosity of the suspension can be higher or lower than the viscosity without the action of the HFEMF. The following cases are possible:

1) $\omega\tau < 1$. The effective shear viscosity increases with increase in HFEMF intensity. This is explained by the fact that at given frequencies, the HFEMF decelerates the rotation of the suspension particles, and this always leads to an increase in viscosity.

2) $\omega\tau = 1$. The viscosity does not depend on the action of the HFEMF.

3) $\omega\tau > 1$. The shear viscosity decreases with increase in HFEMF intensity. Furthermore, there is a critical frequency $\omega_* = \tau^{-1}\sqrt{3}$ at which the effective viscosity is minimal, which indicates a resonant nature of the action. At these frequencies, the HFEMF accelerates the rotation of the suspension particles, which leads to a decrease in viscosity.

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